

Luminosity density estimation from redshift surveys and the mass density of the Universe

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ABSTRACT

In most direct estimates of the mass density (visible or dark) of the Universe, a central input parameter is the luminosity density of the Universe. Here we consider the measurement of this luminosity density from red-shift surveys, as a function of the yet undetermined characteristic scale R_H at which the spatial distribution of visible matter tends to a well defined homogeneity. Making the canonical assumption that the cluster mass to luminosity ratio \mathcal{M}/\mathcal{L} is the universal one, we can estimate the total mass density as a function $\Omega_m(R_H, \mathcal{M}/\mathcal{L})$. Taking the highest estimated cluster value $\mathcal{M}/\mathcal{L} \approx 300hM_\odot/L_\odot$ and a conservative lower limit $R_H \gtrsim 20h^{-1}\text{Mpc}$, we obtain the upper bound $\Omega_m \lesssim 0.1$. We note that for values of the homogeneity scale R_H in the range $R_H \approx (90 \pm 45)h\text{Mpc}$, the value of Ω_m may be compatible with the nucleosynthesis inferred density in baryons.

Subject headings: galaxies: general; galaxies: statistics; cosmology: large-scale structure of the universe

While the density of visible matter can be directly inferred from red-shift surveys, that of dark matter is necessarily only more indirectly accessible. The currently most popular methods for determining the latter make use of estimates of the total mass of clusters, which are stated as estimates for the mass to luminosity ratio for clusters (see Bahcall 1999 for a review and Hradecky et al. 2000 for recent determinations in groups and clusters of galaxies). Assuming that this ratio is representative of the global ratio of mass to luminosity one can infer, *given the luminosity density ϕ_* of the Universe*, the corresponding value for the mass density Ω_m . Almost invariably in the literature this luminosity density is taken from an analysis reported in Efstathiou et al. (1988), which derives a value ϕ_* primarily by fitting the normalization of number counts from the origin as a function of apparent magnitude. The intrinsic problems of inferring what is a three dimensional property from such two dimensional (projected) measures have been discussed in detail elsewhere (e.g. Sylos Labini, Montuori & Pietronero 1998 -hereafter SLMP98). We limit ourselves to noting here that this procedure of normalization is highly sensitive to the (a priori unknown) corrections which are applied to the data, which without such corrections do not even show a counts' slope consistent with the homogeneity assumption (i.e. $\alpha = 0.6$) in any range of apparent magnitude. Further, despite the quite precise value (error of 20%) derived in Efstathiou et al., the values of ϕ_* found in different surveys in the paper *vary by a factor of four*. The authors note that “the dispersion amongst the estimates is extraordinarily large”, but do not provide any clear explanation of why this is so and how they arrive at such a small error in their final (averaged) estimate for ϕ_* . It is our view that such variations are intrinsic to the data, corresponding to the fact that each result is normalized to samples of different size and geometry. In any case it is clear that the most reliable way of estimating this three dimensional property of the data is directly from three dimensional data, and it is this which we do here.

Before entering the details of data analysis, let us discuss important methodological

point which is at the centre of our treatment: instead of assuming homogeneity in the data sets we analyse, we use statistical methods which do not depend on the presence or absence of such homogeneity. Here this means that instead of assuming that ϕ^* is well defined a priori, we evaluate a more general quantity which will correspond to it in the case that the distribution is homogeneous. Homogeneity at large scales in the distribution of matter is a central assumption of standard cosmological models and of the statistical tools usually used to analyze the data. While two dimensional (angular) maps of galaxies initially provided clear support for the supposition of homogeneity at relatively small scales, three dimensional red-shift surveys revealed unsuspected structure at much larger scales. The existence of such structures (in particular voids) is incompatible with well defined homogeneity below these scales, and observationally the actual scale characterizing homogeneity in matter, and indeed the existence of such a scale at all, has become a subject of considerable debate (e.g SLMP98 and Wu, Lahav & Rees 1999). Standard characterizations of the three dimensional galaxy and cluster red-shift data, which simply assume homogeneity at the largest scale probed, give rise to an ever-growing range of characteristic scales (“correlation lengths” of different classes of objects), while an analysis of the same surveys with methods which do not assume such homogeneity leads to an interpretation in which these scales are sample-dependent characterizations of a distribution with an underlying simple scale invariance, a cut-off from which to homogeneity has, it is argued, not yet been detected with any statistical significance (see e.g. Joyce, Montuori & Sylos Labini 1999 - hereafter JMSL99). Our aim in this paper is not to address these issues which are discussed in detail elsewhere, but to show how crucial input parameters to standard cosmological models depend on what are, at the very least, important observational uncertainties concerning the distribution of visible matter. In particular we consider here the total mass density of the Universe Ω_m , but the same kind of analysis can be easily applied to other important parameters (for example, to the amplitude of the matter power spectrum). We express our

results in terms of a well defined homogeneity scale to be determined from red-shift surveys and place bounds on Ω_m corresponding to conservative current lower bounds on this scale.

We first consider the properties of the spatial distribution of visible matter and the characterization of the tendency (if any) to homogeneity. At small scales, at least up to $\sim 20h^{-1}$ Mpc, it is widely agreed that the galaxy distribution shows fractal behavior. Deviation away from this behavior towards the expected homogeneity is most easily identified using a very simple two point statistic, the average conditional density

$$\Gamma(r) = \left\langle \frac{1}{S(r)} \frac{dN(< r)}{dr} \right\rangle_p \quad (1)$$

where $dN(< r)$ is the number of points in a shell of thickness dr at distance r from an *occupied point* and $S(r)dr$ is the volume of the shell. The symbol $\langle \dots \rangle_p$ indicates that the quantity is a conditional one (for a discussion see Gabrielli & Sylos Labini 2001), the average being performed over occupied points. This statistic is simply an unnormalized form of the standard two point correlation functions $\xi(r)$ (SLMP98). While homogeneity corresponds to the convergence of Γ to a fixed value as a function of distance, scale-invariance is indicated by the continuation of a simple power law behavior. Only in the former case does a real average density exist. The small scale fractal behaviour observed in red-shift catalogues corresponds to the behaviour $\Gamma(r) = Ar^{D-3}$ (with A a constant); detecting homogeneity corresponds to making an estimate of the asymptotic value of the density $\Gamma_\infty > 0$; we then define $AR_H^{D-3} = \Gamma_\infty$ i.e. R_H is defined as the scale at which the small scale fractal behaviour would match onto the asymptotic density, in the case that there were a simple cross-over from fractality to homogeneity. Once R_H has been defined, one can meaningfully study correlation properties of fluctuations about the mean density Γ_∞ with the usual normalized correlation function $\xi(r) = (\Gamma(r)/\Gamma_\infty) - 1$. The relationship of the scale R_H so defined to other characteristic scales often used is simple to derive. For example, consider the “correlation length” r_o defined by $\xi(r_o) = 1$. If we assume that r_o lies in the range in

which the distribution is well approximated as fractal - which is generally taken to be the case - one finds $r_o = 2^{1/D-3}R_H$. For any particular smooth form of the cross-over scale from small scale fractality to homogeneity the precise relation will be slightly modified.

We now turn to the estimation of the luminosity density from three dimensional surveys. We adapt here the approximation which is always made in this context: We assume that the spatial correlations in galaxy positions are unconnected to their morphological or luminosity properties. While such an assumption is known to be strictly false (Binggeli et al. 1988) - it is inconsistent with local morphological properties (e.g. elliptical galaxies are mostly located in the center of rich clusters (Dressler 1984), and there is a correlation between luminosity and space distributions as discussed in SLMP98) - we will check that it is quantitatively a good approximation for the estimates being made here. With this assumption we can write the factorized expression

$$\langle \nu(r, L) \rangle_p dL d^3r = \phi(L) \Gamma(r) dL d^3r = A r^{D-3} L^\alpha e^{-\frac{L}{L_*}} d^3r dL \quad (2)$$

for the (conditional) average number of galaxies in the volume element d^3r at distance r from a observer located on a galaxy, and with luminosity in the range $[L, L + dL]$. In the latter form we have used the fact that the galaxy luminosity function has been observed to have the so-called Schechter shape with parameters L_* (luminosity cut-off) and α (power law index) which can be determined experimentally (Binggeli et al. 1988), and we have written the small scale fractal behavior for the spatial distribution. Hence $\langle \nu(r, L) \rangle_p$ is a function of the measurable parameters L_*, α and those characterizing the spatial distribution - D, A at small scales and, in the case of detected homogeneity, R_H at large scales. Note that for the determination of the shape of the luminosity function the effect of space inhomogeneities can be neglected if the joint distribution can be written as in Eq.2 (Binggeli et al. 1988, SLMP98). There are different methods to estimate the parameters M_* and α but *all* are based on the assumption embodied in Eq.2: these so-called

inhomogeneity-independent methods have been developed to determine the shape of the luminosity function, independently of its overall normalization. It is now simple to estimate the average luminosity density in a sphere of radius R and volume $V(R)$ placed around a galaxy

$$\langle j(< R) \rangle_p = \frac{1}{V(R)} \int_0^R \int_0^\infty \langle \nu(r, L) \rangle_p L dL d^3r \quad (3)$$

which has the R dependence which follows from that of the space density, with a corresponding asymptotic value in the case of homogeneity. In a volume limited sample (hereafter VL - see e.g. JMSL99) extracted from a given redshift survey, we may compute the number of galaxies as a function of distance. Using Eq. 2 we have

$$\langle N(L > L_{VL}; r < R) \rangle_p = \int_0^R \int_{L_{VL}}^\infty \langle \nu(r, L) \rangle_p dL d^3r = B_{VL} R^D \quad (4)$$

where B_{VL} is the amplitude of the number counts in a VL sample with faint luminosity limit at $L = L_{VL}$. From Eq. 2 and Eq. 4 and considering Eq.3 we then obtain that

$$\langle j(< R) \rangle_p \equiv j(10) \left(\frac{R}{10h^{-1}} \right)^{D-3}, \quad (5)$$

in $L_\odot \cdot Mpc^{-3}$, where $j(10) = 3/(4\pi)L_*(10h^{-1})^{D-3}\gamma(\alpha+2)\Phi_{VL}$ where $\gamma(\alpha+2)$ is the Euler function ($\gamma(n) = (n-1)!$ for positive integers n)¹ and we have defined $\Phi_{VL} = B_{VL}/(\int_{y_{VL}}^\infty y^\alpha e^{-y} dy)$ where $y_{VL} = L_{VL}/L_*$.

¹Note that for the values relevant here $\alpha \approx -1$ the integral in the denominator of Φ_{VL} is a cut-off divergent gamma function ($n < 0$), and depends sensitively on the lower cut off y_{VL} . On the other hand the gamma function is convergent, so that the total luminosity is dominated by galaxies with luminosity $\sim L_*$, and is essentially insensitive to the lower cut off in the luminosity function L_{min} . If there are very many additional very low surface brightness galaxies which are not sampled in redshift surveys, sufficient to make the exponent $\alpha < -2$, this integral would be strongly dependent on L_{min} (and the total luminosity dominated by these faint galaxies).

Employing our definition of the homogeneity scale R_H we obtain the asymptotic average luminosity density by simply substituting $R = R_H$ in Eq.5. Given a value of (or lower bound on) this scale it is thus straightforward to obtain the corresponding value (or upper bound on) the total mass density, once one has an appropriate estimate of the global mass to luminosity ratio. The numerical results we quote here we obtain from the CfA2-South survey (Huchra et al., 1999), which covers a solid angle of about one steradian, with a completeness in its observing range of over 99% and a total of 4392 galaxies. We have repeated our calculations (Montuori et al. 2001) in the larger joint catalogue of CfA2 and SSRS2, including both the Southern and Northern galactic caps, and find results which are in good agreement with those given here. For the luminosity parameters we take $M_*^{ZW} = -18.8 \pm 0.3$ and $\alpha = -1.0 \pm 0.2$ (Marzke, Huchra & Geller 1994). Note that we compute all the relevant quantities in the Zwicky magnitude system used in these surveys. The \mathcal{M}/\mathcal{L} results from clusters refer to luminosity in the B magnitude system, to which the transformation from the Zwicky system is not exactly known (Marzke, Huchra & Geller 1994). In practice, up to a small residual effect due to galaxy type, the transformation should be well modeled as a simple zero point offset $M^B = M^{ZW} + \Delta$ with $\Delta \leq 0.3$ (Paturel et al., 1994). In our estimated luminosity density this induces the correction $j(10)^B = j(10) \times 10^{-0.4\Delta}$, which is small, and we will simply neglect it in what follows. For the spatial properties the results we quote are for the analysis of the CfA2 survey described in JMSL99, using exactly the methods used there to estimate the appropriate parameters. In Table 1 are given the values of B_{VL} determined in different VL samples, defined by the corresponding absolute magnitude limit M_{VL} . The results depend on the cosmological parameters assumed in the reconstruction of distances and absolute magnitudes from redshifts and apparent magnitude. The values quoted correspond to the Mattig relation with $q_0 = 0.5$, but the results do not sensibly change for any other reasonable choice of q_0 as the redshifts involved are very small ($z \leq 0.05$). From B_{VL}

we have computed the quoted values of the quantity Φ_{VL} in the different VL samples, and we infer the average value $\langle \Phi_{VL} \rangle = 1.4 \pm 0.4$. Using this we obtain the numerical value $j(10) \approx (2 \pm 0.6) \times 10^8 h L_{\odot} / Mpc^3$. The fractal dimension D is given by the slope of $\langle N(< r) \rangle_p$ as a function of r in a VL sample. As mentioned above our analysis of the CfA2+SSRS2 joint catalogue gives values very consistent with these. Hereafter we adopt for simplicity the value $D = 2.0$.

For a given R_H we now find the mass density parameter in units of the critical density $\rho_c = 2.78 \cdot 10^{11} h^2 M_{\odot} / Mpc^3$ where M_{\odot} is the solar mass, and as a function of a specified global mass-to-luminosity ratio (in solar and h units), to be

$$\Omega_m \left(R_H, \frac{\mathcal{M}}{\mathcal{L}} \right) = [(6 \pm 2) \times 10^{-4}] \frac{\mathcal{M}}{\mathcal{L}} h^{-1} \left(\frac{10 h^{-1}}{R_H} \right). \quad (6)$$

Note that because estimates of \mathcal{M}/\mathcal{L} are linearly dependent on h , and R_H is measured in units of h^{-1} Mpc, Eq.6 is in fact independent of the Hubble constant.

Before proceeding to discuss this estimate of Ω_m in more detail we comment on the variation in Φ_{VL} seen in Table 1. These fluctuations can be due to one or more of the following factors: (i) Errors (statistical and/or systematic) in the measurement of B_{VL} : this effect can be very important for the samples $M_{VL} < M_*$, as in this range of magnitudes the statistics of the VL samples is much weaker because of the exponential break in the luminosity function. Further, as discussed in various papers (i.e. Bothun & Cornell 1990), the magnitudes in the GCGC catalogue (from which the photometry of CfA2 comes) are based on the Zwicky system, with an estimated error of 0.3^m up to 15.0^m increasing up to $\gtrsim 0.5^m$ for the faint end of the catalogue (i.e. for $15.0 \lesssim m \lesssim 15.7$). The effect of this systematic error is not crucial in the estimation of $\Gamma(r)$ if the statistics is robust (SLMP98), and in the VL considered there is a good spread of absolute magnitudes (that is $M_{VL} \gtrsim -19.5$). Clearly in the deepest and more luminous VL samples ($M_{VL} \lesssim -20.0$) the effect of the photometry error is more important in the determination of the amplitude

of the conditional average density, in view of Malmquist bias (e.g. Teerikorpi 1997).
(ii) Use of non optimal parameters α and M_* in the computation of Φ_{VL} . In order to check the dependence on these two parameters we have let them vary in the range $-1.2 \leq \alpha \leq -0.9$ and $-18.7 \leq M_* \leq -19.1$. In the VL samples with $M_{VL} > M_*$ we do not see a large fluctuation of Φ_{VL} , while for the brighter samples it indeed can cause a change by a substantial factor $10\% \div 30\%$. The values we have used here correspond to $M_* = -19.1$ which give the stablest result for Φ_{VL} . (iii) The breakdown of the assumption of luminosity/space independence embodied in Eq.1: The independence of the determined parameters of the luminosity sample is our consistency test of this assumption, and to the extent that the fluctuations are relatively small it is good. Further given the first two points which may explain much of the observed spread the error caused by this is certainly at most of the order of the 20% we have given.

Let us now consider further our estimate of Ω_m . Taking first the estimate $\mathcal{M}/\mathcal{L} \approx 10h$ in the B -band as derived by Faber and Gallagher (1979), which corresponds to a global mass to luminosity ratio typical of spiral galaxies, we obtain $\Omega_m(R_H) \approx 6 \times 10^{-3} \left(\frac{10h^{-1}}{R_H} \right)$. With $R_H \approx 10h^{-1}\text{Mpc}$ ($r_o \approx 5h^{-1}\text{Mpc}$) we obtain the value $\Omega_g \approx 6 \cdot 10^{-3}$ of the standard treatment of Peebles (1993). On the other hand we can determine the mass to luminosity ratio which would give a critical mass density Universe. For given R_H we find $\left(\frac{\mathcal{M}}{\mathcal{L}} \right)_{crit} \approx 1600h \left(\frac{R_H}{10h^{-1}} \right)$, so that again the canonically quoted value of $(\mathcal{M}/\mathcal{L})_{crit} \approx 1600h$ corresponds to the homogeneity scale $R_H \approx 10h^{-1}\text{Mpc}$.

Galaxy clusters have been much studied in recent years, and they are believed to probe well the global mass to luminosity ratio, for which the observed value is $(\mathcal{M}/\mathcal{L})_c \approx 300h$ in the B -band (Carlberg et al. 1997, Bahcall 1999, Hradecky et al. 2000). Taking this value we obtain $\Omega_m(R_H) \approx (0.18 \pm 0.06) \left(\frac{10h^{-1}}{R_H} \right)$. The value which results using the same standard value $R_H = 10h^{-1}\text{Mpc}$ is $\Omega_m \approx 0.2$ (Bahcall 1999), which simply means that

the former is the homogeneity scale built into the estimate of the luminosity density from Efstathiou et al. (1988) used as the basis for these estimates. The point of the present paper has been to make the dependence on this scale explicit, and to use its value as estimated from three dimensional surveys. In JMSL99 we placed a lower bound of $R_H \approx 20h^{-1}\text{Mpc}$ on the homogeneity scale, and found no clear statistical evidence for the existence of a cut-off to homogeneity at larger scales. Using this as a *conservative lower bound* on that scale we now obtain the upper bound $\Omega_m \leq 0.1$ on the total mass density. In SLMP98 a strong case has been made for a much larger lower bound of $100 \div 150h^{-1}\text{Mpc}$, based on a combination of cluster catalogues and the LEDA red-shift database. While these results remain controversial (e.g. Wu et al., 2000), and need confirmation from forthcoming larger red-shift surveys (2dF and SSDS), it is interesting to consider the implications of such a finding for determination of Ω_m .

One of the most immediate cosmological implications of the measurement of the mass density comes from the comparison of its value with the standard Big Bang nucleosynthesis (SBBN) limits on the baryon density of the universe, which give (Olive et al. 2000, Tytler et al. 2000) $\Omega_b^{BBN} h^2 \approx 0.019 \pm 0.004$. While this comparison results in the inference of the existence of non-baryonic dark matter in the standard case, one can now view it as providing a possible “window of consistency” for the two values. Using the estimate obtained above (6) we find that for the homogeneity scale $R_H = (0.3 \pm 0.15)\mathcal{M}/\mathcal{L}$ Mpc the dark matter in the Universe can be purely baryonic with its global density satisfying the constraints of SBBN. Conversely an homogeneity scale larger than this value would cause a serious problem for the theory of SBBN. Adopting the value $(\mathcal{M}/\mathcal{L})_c \approx 300h$, we find that gives $R_H = (90 \pm 45)h$ Mpc which, for $h = 0.65$, corresponds to $R_H = (60 \pm 30)$ Mpc, which allows potential compatibility even for values of R_H as small as our *conservative* estimation $R_H = 20h^{-1}\text{Mpc} \approx 30\text{Mpc}$.

Various other methods are commonly used to estimate the mass density of the Universe. One is based on clusters is that obtained by observations which constrain the fraction of hot baryonic X-ray emitting gas to the total mass in clusters (see Bahcall 1999). Given that the rest of the mass may be non-baryonic this gives, when one uses the nucleosynthesis upper bound on Ω_b , an upper bound on the total mass $\Omega_m \leq 0.3$. Further, taking most of this mass to be non-baryonic one infers a value of Ω_m consistent with the value from the direct estimate. In the present context we note simply that, if the scale R_H is indeed larger than usually implicitly assumed, the total mass density may be much lower and the dark mass in clusters quite consistently be baryonic.

Another source of estimates for the total mass density comes from peculiar velocity flows (see Strauss & Willick 1994). These make use of linear perturbation theory, in which regime one can correlate the peculiar motions in a simple way with the total mass fluctuations, with the overall amplitude depending on the total Ω_m which is then in principle determinable. In practice the problem is that one does not know how the fluctuations in the dominating dark component are related to those in the visible matter, and only by making some extremely simplistic assumptions (e.g. “linear bias”) can one extract a result. A much greater problem is one of principle related to the scale R_H , as it is in fact precisely also the scale which characterizes the validity of a linear regime. If, as we have discussed here, R_H is much larger than the standard assumed value the estimates which have been performed to date are meaningless. To reliably correlate peculiar velocities with the mass distribution much tighter constraints are first needed on the latter, and completely different methods to the standard ones must be used if the regime of non-linearity extends much deeper than usually assumed (Joyce et al. 2000).

In this paper we have described, taking the example of the matter density Ω_m , how crucial parameters in standard type cosmologies are dependent on a scale which has yet

to be determined. The requisite statistically robust detection and characterization of the cross-over to homogeneity and a reliable determination of this scale will be possible with the forthcoming 2dF and SSDS surveys. On the basis of current data we have placed constraints on Ω_m by using the most suitable publically available three dimensional data available for this purpose, the combined CfA2 and SSRS2 surveys. We note that we have assumed, as is usually done, that clusters do indeed give a reliable measure of the global mass to luminosity ratio, and that if this assumption is not correct our results for the estimated parameters will of course not hold.

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Table 1: Values of B_{VL} estimated in volume limited samples in the CfA2 South catalogue. The absolute magnitude cut is at M_{VL}^{ZW} , In the third column is shown the determined value of the parameter Φ_{VL} where we have used $M_* = -19.1$ and $\alpha = -1.00$ as parameters of the luminosity function. N_{VL} is the number of points in the volume-limited sample.

M_{VL}^{ZW}	B_{VL}	Φ_{VL}	N_{VL}
-17.0	1.5 ± 0.1	1.0 ± 0.1	1641
-18.0	0.7 ± 0.1	0.9 ± 0.1	2518
-19.0	0.4 ± 0.05	1.6 ± 0.2	4134
-19.5	0.17 ± 0.02	1.5 ± 0.2	3868
-20.0	0.06 ± 0.01	1.8 ± 0.3	2524